1. **POPULATION GENETICS AND THE HARDY-WEINBERG LAW**

**The Hardy-Weinberg formulas allow scientists to determine whether evolution has occurred. Any changes in the gene frequencies in the population over time can be detected. The law essentially states that if no evolution is occurring, then an equilibrium of allele frequencies will remain in effect in each succeeding generation of sexually reproducing individuals. In order for equilibrium to remain in effect (i.e. that no evolution is occurring) then the following five conditions must be met:**

* 1. **No mutations must occur so that new alleles do not enter the population.**
	2. **No gene flow can occur (i.e. no migration of individuals into, or out of, the population).**
	3. **Random mating must occur (i.e. individuals must pair by chance)**
	4. **The population must be large so that no genetic drift (random chance) can cause the allele frequencies to change.**
	5. **No selection can occur so that certain alleles are not selected for, or against.**

**Obviously, the Hardy-Weinberg equilibrium cannot exist in real life. Some or all of these types of forces all act on living populations at various times and evolution at some level occurs in all living organisms. The Hardy-Weinberg formulas allow us to detect some allele frequencies that change from generation to generation, thus allowing a simplified method of determining that evolution is occurring. There are two formulas that must be memorized:**

**p2 + 2pq + q2 = 1 *and* p + q = 1**

**p = frequency of the dominant allele in the population
q = frequency of the recessive allele in the population**

**p2 = percentage of homozygous dominant individuals
q2 = percentage of homozygous recessive individuals
2pq = percentage of heterozygous individuals**

**Individuals that have aptitude for math find that working with the above formulas is ridiculously easy. However, for individuals who are unfamiliar with algebra, it takes some practice working problems before you get the hang of it. Below I have provided a series of practice problems that you may wish to try out. Note that I have rounded off some of the numbers in some problems to the second decimal place:**

* 1. **PROBLEM #1.**

**You have sampled a population in which you know that the percentage of the homozygous recessive genotype (aa) is 36%. Using that 36%, calculate the following:**

* + 1. **The frequency of the "aa" genotype.**
		2. **The frequency of the "a" allele.**
		3. **The frequency of the "A" allele.**
		4. **The frequencies of the genotypes "AA" and "Aa."**
		5. **The frequencies of the two possible phenotypes if "A" is completely dominant over "a."**
	1. **PROBLEM #2.**

**Sickle-cell anemia is an interesting genetic disease. Normal homozygous individuals (SS) have normal blood cells that are easily infected with the malarial parasite. Thus, many of these individuals become very ill from the parasite and many die. Individuals homozygous for the sickle-cell trait (ss) have red blood cells that readily collapse when deoxygenated. Although malaria cannot grow in these red blood cells, individuals often die because of the genetic defect. However, individuals with the heterozygous condition (Ss) have some sickling of red blood cells, but generally not enough to cause mortality. In addition, malaria cannot survive well within these "partially defective" red blood cells. Thus, heterozygotes tend to survive better than either of the homozygous conditions. If 9% of an African population is born with a severe form of sickle-cell anemia (ss), what percentage of the population will be more resistant to malaria because they are heterozygous (Ss) for the sickle-cell gene?**

* 1. **PROBLEM #3.**

**There are 100 students in a class. Ninety-six did well in the course whereas four blew it totally and received a grade of F. Sorry. In the highly unlikely event that these traits are genetic rather than environmental, if these traits involve dominant and recessive alleles, and if the four (4%) represent the frequency of the homozygous recessive condition, please calculate the following:**

* + 1. **The frequency of the recessive allele.**
		2. **The frequency of the dominant allele.**
		3. **The frequency of heterozygous individuals.**
	1. **PROBLEM #4.**

**Within a population of butterflies, the color brown (B) is dominant over the color white (b). And, 40% of all butterflies are white. Given this simple information, which is something that is very likely to be on an exam, calculate the following:**

* + 1. **The percentage of butterflies in the population that are heterozygous.**
		2. **The frequency of homozygous dominant individuals.**
	1. **PROBLEM #5.**

**A rather large population of Biology instructors have 396 red-sided individuals and 557 tan-sided individuals. Assume that red is totally recessive. Please calculate the following:**

* + 1. **The allele frequencies of each allele.**
		2. **The expected genotype frequencies.**
		3. **The number of heterozygous individuals that you would predict to be in this population.**
		4. **The expected phenotype frequencies.**
		5. **Conditions happen to be really good this year for breeding and next year there are 1,245 young "potential" Biology instructors. Assuming that all of the Hardy-Weinberg conditions are met, how many of these would you expect to be red-sided and how many tan-sided?**
	1. **PROBLEM #6.**

**A very large population of randomly-mating laboratory mice contains 35% white mice. White coloring is caused by the double recessive genotype, "aa". Calculate allelic and genotypic frequencies for this population.**

* 1. **PROBLEM #7.**

**After graduation, you and 19 of your closest friends (lets say 10 males and 10 females) charter a plane to go on a round-the-world tour. Unfortunately, you all crash land (safely) on a deserted island. No one finds you and you start a new population totally isolated from the rest of the world. Two of your friends carry (i.e. are heterozygous for) the recessive cystic fibrosis allele (c). Assuming that the frequency of this allele does not change as the population grows, what will be the incidence of cystic fibrosis on your island?**

* 1. **PROBLEM #8.**

**You sample 1,000 individuals from a large population for the MN blood group, which can easily be measured since co-dominance is involved (i.e., you can detect the heterozygotes). They are typed accordingly:**

|  |  |  |  |
| --- | --- | --- | --- |
| **BLOOD TYPE** | **GENOTYPE** | **NUMBER OF INDIVIDUALS** | **RESULTING FREQUENCY** |
| **M** | **MM** | **490** | **0.49** |
| **MN** | **MN** | **420** | **0.42** |
| **N** | **NN** | **90** | **0.09** |

**Using the data provide above, calculate the following:**

* + 1. **The frequency of each allele in the population.**
		2. **Supposing the matings are random, the frequencies of the matings.**
		3. **The probability of each genotype resulting from each potential cross.**
	1. **PROBLEM #9.**

**Cystic fibrosis is a recessive condition that affects about 1 in 2,500 babies in the Caucasian population of the United States. Please calculate the following.**

* + 1. **The frequency of the recessive allele in the population.**
		2. **The frequency of the dominant allele in the population.**
		3. **The percentage of heterozygous individuals (carriers) in the population.**
	1. **PROBLEM #10.**

**In a given population, only the "A" and "B" alleles are present in the ABO system; there are no individuals with type "O" blood or with O alleles in this particular population. If 200 people have type A blood, 75 have type AB blood, and 25 have type B blood, what are the alleleic frequencies of this population (i.e., what are p and q)?**

* 1. **PROBLEM #11.**

**The ability to taste PTC is due to a single dominate allele "T". You sampled 215 individuals in biology, and determined that 150 could detect the bitter taste of PTC and 65 could not. Calculate all of the potential frequencies.**

* 1. **PROBLEM #12. (You will not have this type of problem on the exam)**

**What allelic frequency will generate twice as many recessive homozygotes as heterozygotes?**

1. **POPULATION GENETICS AND THE HARDY-WEINBERG LAW
ANSWERS TO SAMPLE QUESTIONS**

**Remember the basic formulas:**

**p2 + 2pq + q2 = 1 *and* p + q = 1**

**p = frequency of the dominant allele in the population
q = frequency of the recessive allele in the population
p2 = percentage of homozygous dominant individuals
q2 = percentage of homozygous recessive individuals
2pq = percentage of heterozygous individuals**

* 1. **PROBLEM #1.**

**You have sampled a population in which you know that the percentage of the homozygous recessive genotype (aa) is 36%. Using that 36%, calculate the following:**

* + 1. **The frequency of the "aa" genotype. Answer: 36%, as given in the problem itself.**
		2. **The frequency of the "a" allele. Answer: The frequency of aa is 36%, which means that q2 = 0.36, by definition. If q2 = 0.36, then q = 0.6, again by definition. Since q equals the frequency of the a allele, then the frequency is 60%.**
		3. **The frequency of the "A" allele. Answer: Since q = 0.6, and p + q = 1, then p = 0.4; the frequency of A is by definition equal to p, so the answer is 40%.**
		4. **The frequencies of the genotypes "AA" and "Aa." Answer: The frequency of AA is equal to p2, and the frequency of Aa is equal to 2pq. So, using the information above, the frequency of AA is 16% (i.e. p2 is 0.4 x 0.4 = 0.16) and Aa is 48% (2pq = 2 x 0.4 x 0.6 = 0.48).**
		5. **The frequencies of the two possible phenotypes if "A" is completely dominant over "a." Answers: Because "A" is totally dominate over "a", the dominant phenotype will show if either the homozygous "AA" or heterozygous "Aa" genotypes occur. The recessive phenotype is controlled by the homozygous aa genotype. Therefore, the frequency of the dominant phenotype equals the sum of the frequencies of AA and Aa, and the recessive phenotype is simply the frequency of aa. Therefore, the dominant frequency is 64% and, in the first part of this question above, you have already shown that the recessive frequency is 36%.**
	1. **PROBLEM #2.**

**Sickle-cell anemia is an interesting genetic disease. Normal homozygous individials (SS) have normal blood cells that are easily infected with the malarial parasite. Thus, many of these individuals become very ill from the parasite and many die. Individuals homozygous for the sickle-cell trait (ss) have red blood cells that readily collapse when deoxygenated. Although malaria cannot grow in these red blood cells, individuals often die because of the genetic defect. However, individuals with the heterozygous condition (Ss) have some sickling of red blood cells, but generally not enough to cause mortality. In addition, malaria cannot survive well within these "partially defective" red blood cells. Thus, heterozygotes tend to survive better than either of the homozygous conditions. If 9% of an African population is born with a severe form of sickle-cell anemia (ss), what percentage of the population will be more resistant to malaria because they are heterozygous (Ss) for the sickle-cell gene? Answer: 9% =.09 = ss = q2. To find q, simply take the square root of 0.09 to get 0.3. Since p = 1 - 0.3, then p must equal 0.7. 2pq = 2 (0.7 x 0.3) = 0.42 = 42% of the population are heterozygotes (carriers).**

* 1. **PROBLEM #3.**

**There are 100 students in a class. Ninety-six did well in the course whereas four blew it totally and received a grade of F. Sorry. In the highly unlikely event that these traits are genetic rather than environmental, if these traits involve dominant and recessive alleles, and if the four (4%) represent the frequency of the homozygous recessive condition, please calculate the following:**

* + 1. **The frequency of the recessive allele. Answer: Since we believe that the homozygous recessive for this gene (q2) represents 4% (i.e. = 0.04), the square root (q) is 0.2 (20%).**
		2. **The frequency of the dominant allele. Answer: Since q = 0.2, and p + q = 1, then p = 0.8 (80%).**
		3. **The frequency of heterozygous individuals. Answer: The frequency of heterozygous individuals is equal to 2pq. In this case, 2pq equals 0.32, which means that the frequency of individuals heterozygous for this gene is equal to 32% (i.e. 2 (0.8)(0.2) = 0.32).**
	1. **PROBLEM #4.**

**Within a population of butterflies, the color brown (B) is dominant over the color white (b). And, 40% of all butterflies are white. Given this simple information, which is something that is very likely to be on an exam, calculate the following:**

* + 1. **The percentage of butterflies in the population that are heterozygous.**
		2. **The frequency of homozygous dominant individuals.**

**Answers: The first thing you'll need to do is obtain p and q. So, since white is recessive (i.e. bb), and 40% of the butterflies are white, then bb = q2 = 0.4. To determine q, which is the frequency of the recessive allele in the population, simply take the square root of q2 which works out to be 0.632 (i.e. 0.632 x 0.632 = 0.4). So, q = 0.63. Since p + q = 1, then p must be 1 - 0.63 = 0.37. Now then, to answer our questions. First, what is the percentage of butterflies in the population that are heterozygous? Well, that would be 2pq so the answer is 2 (0.37) (0.63) = 0.47. Second, what is the frequency of homozygous dominant individuals? That would be p2 or (0.37)2 = 0.14.**

* 1. **PROBLEM #5.**

**A rather large population of Biology instructors have 396 red-sided individuals and 557 tan-sided individuals. Assume that red is totally recessive. Please calculate the following:**

* + 1. **The allele frequencies of each allele. Answer: Well, before you start, note that the allelic frequencies are p and q, and be sure to note that we don't have nice round numbers and the total number of individuals counted is 396 + 557 = 953. So, the recessive individuals are all red (q2) and 396/953 = 0.416. Therefore, q (the square root of q2) is 0.645. Since p + q = 1, then p must equal 1 - 0.645 = 0.355.**
		2. **The expected genotype frequencies. Answer: Well, AA = p2 = (0.355)2 = 0.126; Aa = 2(p)(q) = 2(0.355)(0.645) = 0.458; and finally aa = q2 = (0.645)2 = 0.416 (you already knew this from part A above).**
		3. **The number of heterozygous individuals that you would predict to be in this population. Answer: That would be 0.458 x 953 = about 436.**
		4. **The expected phenotype frequencies. Answer: Well, the "A" phenotype = 0.126 + 0.458 = 0.584 and the "a" phenotype = 0.416 (you already knew this from part A above).**
		5. **Conditions happen to be really good this year for breeding and next year there are 1,245 young "potential" Biology instructors. Assuming that all of the Hardy-Weinberg conditions are met, how many of these would you expect to be red-sided and how many tan-sided? Answer: Simply put, The "A" phenotype = 0.584 x 1,245 = 727 tan-sided and the "a" phenotype = 0.416 x 1,245 = 518 red-sided ( or 1,245 - 727 = 518).**
	1. **PROBLEM #6.**

**A very large population of randomly-mating laboratory mice contains 35% white mice. White coloring is caused by the double recessive genotype, "aa". Calculate allelic and genotypic frequencies for this population. Answer: 35% are white mice, which = 0.35 and represents the frequency of the aa genotype (or q2). The square root of 0.35 is 0.59, which equals q. Since p = 1 - q then 1 - 0.59 = 0.41. Now that we know the frequency of each allele, we can calculate the frequency of the remaining genotypes in the population (AA and Aa individuals). AA = p2 = 0.41 x 0.41 = 0.17; Aa = 2pq = 2 (0.59) (0.41) = 0.48; and as before aa = q2 = 0.59 x 0.59 = 0.35. If you add up all these genotype frequencies, they should equal 1.**

* 1. **PROBLEM #7.**

**After graduation, you and 19 of your closest friends (lets say 10 males and 10 females) charter a plane to go on a round-the-world tour. Unfortunately, you all crash land (safely) on a deserted island. No one finds you and you start a new population totally isolated from the rest of the world. Two of your friends carry (i.e. are heterozygous for) the recessive cystic fibrosis allele (c). Assuming that the frequency of this allele does not change as the population grows, what will be the incidence of cystic fibrosis on your island?Answer: There are 40 total alleles in the 20 people of which 2 alleles are for cystic fibrous. So, 2/40 = .05 (5%) of the alleles are for cystic fibrosis. That represents p. Thus, cc or p2 = (.05)2 = 0.0025 or 0.25% of the F1 population will be born with cystic fibrosis.**

* 1. **PROBLEM #8.**

**You sample 1,000 individuals from a large population for the MN blood group, which can easily be measured since co-dominance is involved (i.e., you can detect the heterozygotes). They are typed accordingly:**

|  |  |  |  |
| --- | --- | --- | --- |
| **BLOOD TYPE** | **GENOTYPE** | **NUMBER OF INDIVIDUALS** | **RESULTING FREQUENCY** |
| **M** | **MM** | **490** | **0.49** |
| **MN** | **MN** | **420** | **0.42** |
| **N** | **NN** | **90** | **0.09** |

**Using the data provide above, calculate the following:**

* + 1. **The frequency of each allele in the population. Answer: Since MM = p2, MN = 2pq, and NN = q2, then p (the frequency of the M allele) must be the square root of 0.49, which is 0.7. Since q = 1 - p, then q must equal 0.3.**
		2. **Supposing the matings are random, the frequencies of the matings. Answer: This is a little harder to figure out. Try setting up a "Punnett square" type arrangement using the 3 genotypes and multiplying the numbers in a manner something like this:**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **MM (0.49)** | **MN (0.42)** | **NN (0.09)** |
| **MM (0.49)** | **0.2401\*** | **0.2058** | **0.0441** |
| **MN (0.42)** | **0.2058** | **0.1764\*** | **0.0378** |
| **NN (0.09)** | **0.0441** | **0.0378** | **0.0081\*** |

* + 1. **Note that three of the six possible crosses are unique (\*), but that the other three occur twice (i.e. the probabilities of matings occurring between these genotypes is TWICE that of the other three "unique" combinations. Thus, three of the possibilities must be doubled.**
		2. **MM x MM = 0.49 x 0.49 = 0.2401
		MM x MN = 0.49 x 0.42 = 0.2058 x 2 = 0.4116
		MM x NN = 0.49 x 0.09 = 0.0441 x 2 = 0.0882
		MN x MN = 0.42 x 0.42 = 0.1764
		MN x NN = 0.42 x 0.09 = 0.0378 x 2 = 0.0756
		NN x NN = 0.09 x 0.09 = 0.0081**
		3. **The probability of each genotype resulting from each potential cross. Answer: You may wish to do a simple Punnett's square monohybrid cross and, if you do, you'll come out with the following result:**

**MM x MM = 1.0 MM
MM x MN = 0.5 MM 0.5 MN
MM x NN = 1.0 MN
MN x MN = 0.25 MM 0.5 MN 0.25 NN
MN x NN = 0.5 MN 0.5 NN
NN x NN = 1.0 NN**

* 1. **PROBLEM #9.**

**Cystic fibrosis is a recessive condition that affects about 1 in 2,500 babies in the Caucasian population of the United States. Please calculate the following.**

* + 1. **The frequency of the recessive allele in the population. Answer: We know from the above that q2 is 1/2,500 or 0.0004. Therefore, q is the square root, or 0.02. That is the answer to our first question: the frequency of the cystic fibrosis (recessive) allele in the population is 0.02 (or 2%).**
		2. **The frequency of the dominant allele in the population. Answer: The frequency of the dominant (normal) allele in the population (p) is simply 1 - 0.02 = 0.98 (or 98%).**
		3. **The percentage of heterozygous individuals (carriers) in the population. Answer: Since 2pq equals the frequency of heterozygotes or carriers, then the equation will be as follows: 2pq = (2)(.98)(.02) = 0.04 or 1 in 25 are carriers.**
	1. **PROBLEM #10.**

**In a given population, only the "A" and "B" alleles are present in the ABO system; there are no individuals with type "O" blood or with O alleles in this particular population. If 200 people have type A blood, 75 have type AB blood, and 25 have type B blood, what are the alleleic frequencies of this population (i.e., what are p and q)? Answer: To calculate the allele frequencies for A and B, we need to remember that the individuals with type A blood are homozygous AA, individuals with type AB blood are heterozygous AB, and individuals with type B blood are homozygous BB. The frequency of A equals the following: 2 x (number of AA) + (number of AB) divided by 2 x (total number of individuals). Thus 2 x (200) + (75) divided by 2 (200 + 75 + 25). This is 475/600 = 0.792 = p. Since q is simply 1 - p, then q = 1 - 0.792 or 0.208.**

* 1. **PROBLEM #11.**

**The ability to taste PTC is due to a single dominate allele "T". You sampled 215 individuals in biology, and determined that 150 could detect the bitter taste of PTC and 65 could not. Calculate all of the potential frequencies. Answer: First, lets go after the recessives (tt) or q2. That is easy since q2 = 65/215 = 0.302. Taking the square root of q2, you get 0.55, which is q. To get p, simple subtract q from 1 so that 1 - 0.55 = 0.45 = p. Now then, you want to find out what TT, Tt, and tt represent. You already know that q2 = 0.302, which is tt. TT = p2 = 0.45 x 0.45 = 0.2025. Tt is 2pq = 2 x 0.45 x 0.55 = 0.495. To check your own work, add 0.302, 0.2025, and 0.495 and these should equal 1.0 or very close to it. This type of problem may be on the exam.**

* 1. **PROBLEM #12. (You will not have this type of problem on the exam)**

**What allelic frequency will generate twice as many recessive homozygotes as heterozygotes? Answer: We need to solve for the following equation: q2 (aa) = 2 x the frequency of Aa. Thus, q2 (aa) = 2(2pq). Or another way of writing it is q2 = 4 x p x q. We only want q, so lets trash p. Since p = 1 - q, we can substitute 1 - q for p and, thus, q2 = 4 (1 - q) q. Then, if we multiply everything on the right by that lone q, we get q2 = 4q - 4q2. We then divide both sides through by q and get q = 4 - 4q. Subtracting 4 from both sides, and then q (i.e. -4q minus q = -5q) also from both sides, we get -4 = -5q. We then divide through by -5 to get -4/-5 = q, or anotherwards the answer which is 0.8 =q. I cannot imagine you getting this type of problem in this general biology course although if you take algebra good luck.**